Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Monday 13 January 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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P 7 9 5 9 1 A 0 1 3 2

Pearson

1. The arithmetic series S is given by

$$S = 2 + 5 + 8 + 11 + \dots\dots\dots + 254$$

Find

(a) the number of terms in the series,

(2)

(b) the sum of the series.

(2)

a) We can use the arithmetic sequence nth term formula to work this out

$A_n = A + (n-1)d$

$A = 2$, common difference $= 8-5 = 3$

Arithmetic Sequence: $A_n = A + (n-1)d$

$\therefore 254 = 2 + (n-1)3$

$254 = 2 + 3n - 3$

$254 = 3n - 1$

$255 = 3n$

$n = 85$

$\therefore$ no. of terms $= 85$

b) As we know our nth term now, we can use the arithmetic sum of series formula

$S_n = \frac{n}{2}[2a + (n-1)d]$

Sum of Arithmetic Series: $S_n = \frac{n}{2}[2a + (n-1)d]$

$\frac{85}{2}[2(2) + (85-1)3]$

$= 10880$

$\therefore$ Sum of the series $= 10880$

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Question 1 continued

(Total for Question 1 is 4 marks)

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 5x)^8$$

giving each term in simplest form.

(4)

This expansion is to be used to find an approximation for 2.05^8

(b) State the value of x that should be used.

(There is no need to carry out this calculation.)

(1)

a) We can use the binomial series formula to work this out:

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$(2-5x)^8 = 2^8 + {}^8C_1(2)^{8-1}(-5x)^1 + {}^8C_2(2)^{8-2}(-5x)^2 + {}^8C_3(2)^{8-3}(-5x)^3$$

$$= 256 + 8(128)(-5x) + 28(64)(25x^2) + 56(32)(-125x^3)$$

$$\therefore = 256 - 5120x + 44800x^2 - 224000x^3 \ldots$$

b) To work out the value of x we can equate $2-5x$ and 2.05

$2 - 5x = 2.05$

$\therefore 5x = -0.05$

$x = -0.01$

$\therefore x = -0.01$

Question 2 continued

(Total for Question 2 is 5 marks)

3.

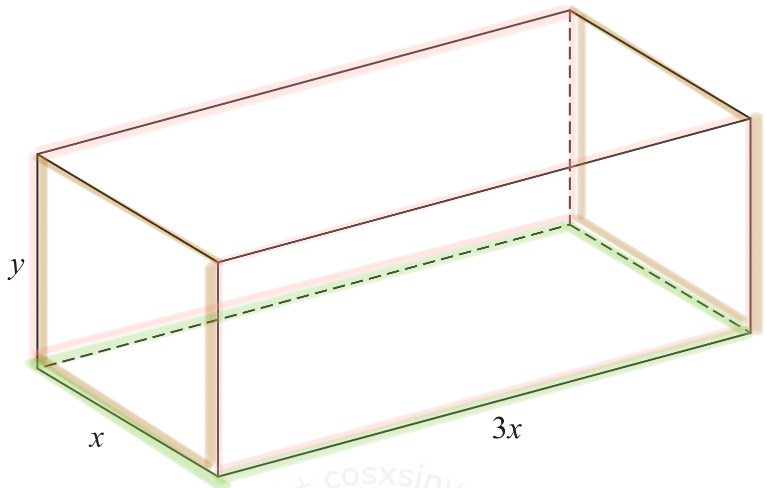

Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

$\frac{dy}{dx} = 0$

(4)

(c) Find $\frac{d^2A}{dx^2}$ and hence show that the value of x found in part (b) gives the minimum value of A.

2nd derivative

(2)

a) ① We know the capacity (which is volume) = 120, so let's form an equation for this

$120 = 3x \times x \times y$

$120 = 3x^2 y$

$\therefore y = \frac{40}{x^2}$

② Now let's find the equation for the surface area (with no top)

Base Area $= 3x \times x = 3x^2$

2 long sides $= 2(3x \times y) = 6xy$

Question 3 continued

2 Short sides = $2(x \times y) = 2xy$

$\therefore$ Total area $= 3x^2 + 6xy + 2xy$

$A = 3x^2 + 8xy$

③ As we have worked y $\left(\frac{40}{x^2}\right)$, we can substitute this into our total surface area equation

$A = 3x^2 + 8x\left(\frac{40}{x^2}\right)$

$A = 3x^2 + \frac{320}{x}$

$\therefore A = 3x^2 + \frac{320}{x}$

where $P = 3$ and $Q = 320$

b) ① The stationary point is when the derivative = 0

$A = 3x^2 + \underbrace{\frac{320}{x}}$

$\frac{dA}{dx} =$ can be written as: $320x^{-1}$

$\therefore A = 3x^2 + 320x^{-1}$

$\frac{dA}{dx} = 6x - 320x^{-2}$

$\downarrow$ rewritten

$\frac{dA}{dx} = 6x - \frac{320}{x^2}$

② We can now equate the derivative to 0 and then solve for x

$0 = 6x - \frac{320}{x^2}$

$\frac{320}{x^2} = 6x$

$6x^3 = 320$

$x^3 = \frac{160}{3}$

$x = \sqrt[3]{\frac{160}{3}} = 3.76$

$\therefore$ Stationary Point $= 3.76$

Question 3 continued

c) ① We can differentiate our part b again to find the second derivative

$$\frac{dA}{dx} = 6x - \frac{320}{x^2}$$

$$\downarrow$$

$$6x - 320x^{-2}$$

$$\therefore \frac{d^2A}{dx^2} = 6 + 640x^{-3}$$

$$\downarrow$$

$$6 + \frac{640}{x^3}$$

Since $x > 0$, we can say that both terms are positive:

$\therefore \frac{d^2A}{dx^2} > 0$ $\therefore$ is a minimum

Question 3 continued

(Total for Question 3 is 10 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that, in a particular geometric series,

- the sum of the first three terms is 70.2
- the sum to infinity is 75

find, for this series,

(a) the common ratio,

(4)

(b) the first term.

(2)

a) ① Let's form our equation for the sum of the first 3 terms

$S_n = 70.2$ and $n = 3$

$$70.2 = \frac{a(1-r^3)}{1-r} \rightarrow ①$$

Sum of Geometric Series:

$$S_n = \frac{a(1-r^n)}{1-r}$$

② And then our equation for sum to infinity

$$75 = \frac{a}{1-r} \rightarrow ②$$

Sum to Infinity:

$$S_\infty = \frac{a}{1-r}$$

for $|r| < 1$

③ To find the common ratio, we can substitute in equation 2 into equation 1

$$70.2 = 75(1-r^3)$$

$$70.2 = 75 - 75r^3$$

$$75r^3 = 4.8$$

$$r = 0.4$$

$\therefore$ common ratio $= 0.4$

* a = first term

* r = common difference

b) To find the first term (a), we can just substitute our common ratio (r) into the sum to infinity equation to work out a

$$75 = \frac{a}{1-0.4}$$

$\therefore a = 45$

$\therefore$ first term $= 45$

P 7 9 5 9 1 A 0 1 0 3 2

Question 4 continued

(Total for Question 4 is 6 marks)

5. $$f(x) = 3x^3 + ax^2 - 10x + b$$

where a and b are constants.

Given that $(3x - 4)$ is a factor of $f(x)$,

$\hookrightarrow f(x)=0$

(a) show that $16a + 9b = 56$

(2)

Given further that when $f(x)$ is divided by $(x - 2)$ the remainder is b,

(b) find the value of a and the value of b.

(4)

(c) Hence, using algebra, fully factorise $f(x)$.

(3)

a) As $(3x-4)$ is a factor of $f(x)$, the x value will result in $f(x)=0$

$(3x-4) \rightarrow x = \frac{4}{3}$

$\therefore f\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^3 + a\left(\frac{4}{3}\right)^2 - 10\left(\frac{4}{3}\right) + b$

$(\times 9) \quad 0 = \frac{64}{9} + \frac{16}{9}a - \frac{40}{3} + b \quad \times 9$

$\therefore 0 = 64 + 16a - 120 + 9b$

$\therefore 16a + 9b = 56$

Thus proven

b) ① We can set x=2 and substitute this into our f(x) equation

$f(2) = b$

$\cancel{b} = 3(2)^3 + a(2)^2 - 10(2) + \cancel{b}$

$\therefore 4a = -4$

$a = -1$

② Now substitute in a=-1 into $16a+9b=56$

$16(-1) + 9b = 56$

$-16 + 9b = 56$

$9b = 72$

$\therefore b = 8$

$\therefore b = 8$

Question 5 continued

c) 1 Let's rewrite f(x) with the correct values of a and b

$f(x) = 3x^3 - x^2 - 10x + 8$

2 Now we can factorise by taking (3x-4) out of the equation f(x)

$f(x) = 3x^3 - x^2 - 10x + 8$

$= (3x-4)\underbrace{(x^2+x-2)}_{(x+2)(x-1)}$

factorise further

$\therefore$ Factors $= (3x-4)(x+2)(x-1)$

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6.

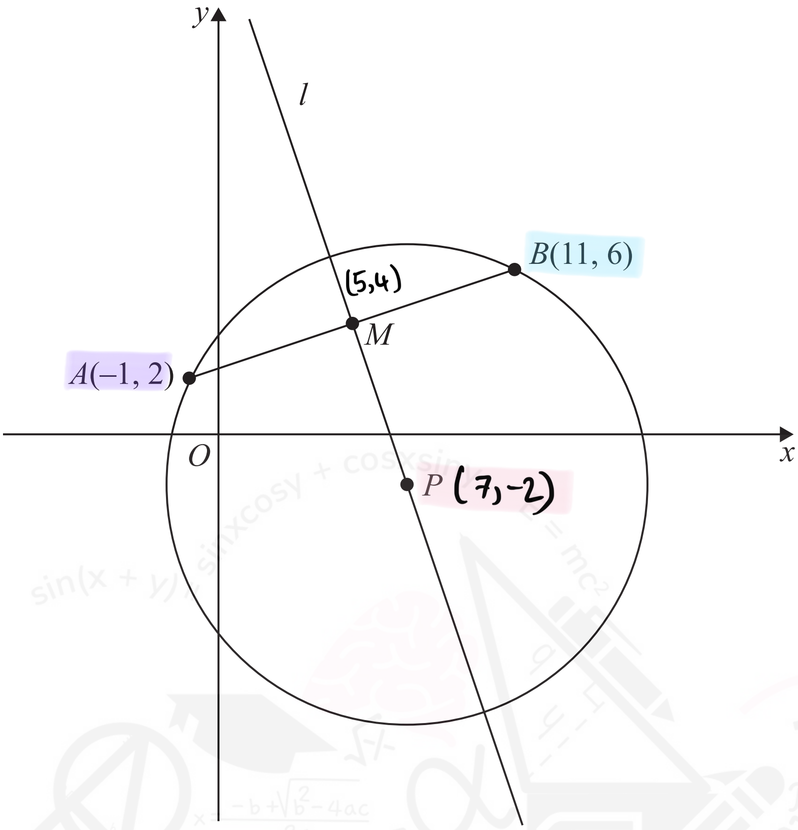

Figure 2

The point A (–1, 2) and the point B (11, 6) both lie on a circle with centre P.

The point M is the midpoint of AB.

Given that the line l passes through M and P, as shown in Figure 2,

(a) find an equation for l, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Given that P has coordinates (7, k), where k is a constant,

(b) find the value of k,

(1)

(c) find an equation for the circle.

(3)

a) ① First let's find point M (which is the midpoint of points A and B)

Point M x coordinate $= \dfrac{-1+11}{2} = 5$

y coordinate $= \dfrac{2+6}{2} = 4$

$\therefore$ Point $M = (5,4)$

Question 6 continued

(2) As line l is perpendicular to AB, we can find the gradient of AB and therefore the gradient of line l

$$= \frac{6+2}{11+(-1)}$$

$$= \frac{1}{3}$$

Gradient:

$m = \dfrac{\text{change in } y}{\text{change in } x}$

$\therefore$ Gradient of line l = negative reciprocal

$\therefore -3$

(3) We can now find the equation of the line using y=mx+c with point M

$y = mx + c$

$4 = (-3)5 + c$

$4 = -15 + c$

$\therefore c = 19$

Line l $\therefore$ $y = -3x + 19$

b) As Point P is on line l, we can substitute in the x value of P into our equation of line l which we found above

Point $P = (7, k)$ Line l: $y = -3x + 19$

$\therefore y = -3(7) + 19$

$= -2$

$\therefore$ Point $P = (7, -2)$ where $k = -2$

c) (1) We need to first find the radius using Point P (which is the centre of the circle) and Point B (which lies on the circle)

Radius using Pythagoras:

$r^2 = (11-7)^2 + (6--2)^2$

$r^2 = 80$

Question 6 continued

(2) We can use the equation of a circle equation now as we know the radius and midpoint P

Point $P = (7, -2)$ and $r^2 = 80$

$(x-7)^2 + (y--2)^2 = 80$

Equation of a Circle:
$(x-a)^2 + (y-b)^2 = r^2$

∴ Equation of the circle:

$(x-7)^2 + (y+2)^2 = 80$

Question 6 continued

(Total for Question 6 is 8 marks)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The table below shows values of x and y, where $y = \log_{10}(x+5)$, for x values between -1 and 4

x	-1	0	1	2	3	4
$y = \log_{10}(x+5)$	$\log_{10}4$	$\log_{10}5$	$\log_{10}6$	$\log_{10}7$	$\log_{10}8$	$\log_{10}9$

Using the trapezium rule with all the y values in the given table, show that

$$\int_{-1}^{4} \log_{10}(x+5)\,dx \approx \log_{10} k$$

where k is an integer to be found.

(3)

(ii) Find the value of a such that

$$2\log_5(5-a) - \log_5(a+25) = 1$$

(5)

i) ① We just need to substitute our y values into the trapezium rule formula

Trapezium Rule:

$\int_a^b y\,dx \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \ldots)\right]$

where $h = \frac{b-a}{n}$

$h = \frac{4-(-1)}{5} = 1$

↳ there will be 5 trapeziums or 5 strips

$\therefore \frac{1}{2}(1)\left[\log 4 + \log 9 + 2(\log 5 + \log 6 + \log 7 + \log 8)\right]$

② Now we just need to simplify and solve

$= \frac{1}{2}\log(4\times 9) + \log(5\times 6\times 7\times 8)$

$= \log 10080$

$\therefore = \log 10080$

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Question 7 continued

ii) ① Use log laws to rewrite our equation

Log Laws:

$\log_b(x^n) = n\log_b(x)$

$\log_b(A) - \log_b(B) = \log_b\left(\frac{A}{B}\right)$

$\log_b(x) = y \leftrightarrow b^y = x$

$2\log_5(5-a) - \log_5(a+25) = 1$

$\log_5((5-a)^2) - \log_5(a+25) = 1$

Rewritten as: $\log_5\left(\frac{(5-a)^2}{(a+25)}\right) = 1$

② Remove the logarithm

$\log_5\left(\frac{(5-a)^2}{(a+25)}\right) = 1$

$\frac{(5-a)^2}{(a+25)} = 5^1 = 5$

$\therefore \frac{(5-a)^2}{(a+25)} = 5$

③ Now we can solve algebriacally to find the value of a

$\therefore \frac{(5-a)^2}{(a+25)} = 5$

$(5-a)^2 = 5(a+25)$

$25 - 10a + a^2 = 5a + 125$

$a^2 - 15a - 100 = 0$

$(a-20)(a+5) = 0$

$\therefore a = 20$ or $a = -5$

④ For any logarithm, log(x) is only defined if x>0

$\therefore$ In the first log:

$5 - a > 0$

$\therefore a < 5$

In the second log:

$a + 25 > 0$

$\therefore a > -25$

Combined:

$-25 < a < 5$

allowed range of solutions

Question 7 continued

(5) Now we need to apply the check ($-25 < a < 5$) to our values of a

when $a = 20$ ✗ too big for range

when $a = -5$ ✓ fits the range

$\therefore a = -5$

Question 7 continued

(Total for Question 7 is 8 marks)

8. (i) A student states

"If x and y are irrational numbers, $x \neq y$, then xy is also irrational."

Show, by counter example, that this statement is not always true.

(1)

(ii) Prove, using algebra, that for all odd integers n, the value of the expression

$$n^3 + 3n + 2$$

is always even but never a multiple of 4

(4)

i) To prove by counter example, we just have to find a value of x that does not follow what the student says

Irrational numbers = surds

and when $x \neq y$

$\therefore$ lets say $x = \sqrt{2}$ and $y = \sqrt{8}$

$\therefore xy = \sqrt{2} \times \sqrt{8}$

$= \sqrt{16}$

$= 4$ which is a rational number $\therefore$ The statement is not always true

ii) Lets call odd numbers as 2k+1 and then substitute this into our given equation

$n = 2k+1$

$\therefore n^3 + 3n + 2$

$(2k+1)^3 + 3(2k+1) + 2$

$\therefore = 8k^3 + 12k^2 + 12k + 6$

We can also rewrite this as the following to prove that we have an even value but not a multiple of 4

$8k^3 + 12k^2 + 12k + 6$

$= 4(2k^3 + 3k^2 + 3k + 1) + 2$

↳ which is even but not a multiple of 4

Hence proven

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Question 8 continued

(Total for Question 8 is 5 marks)

9.

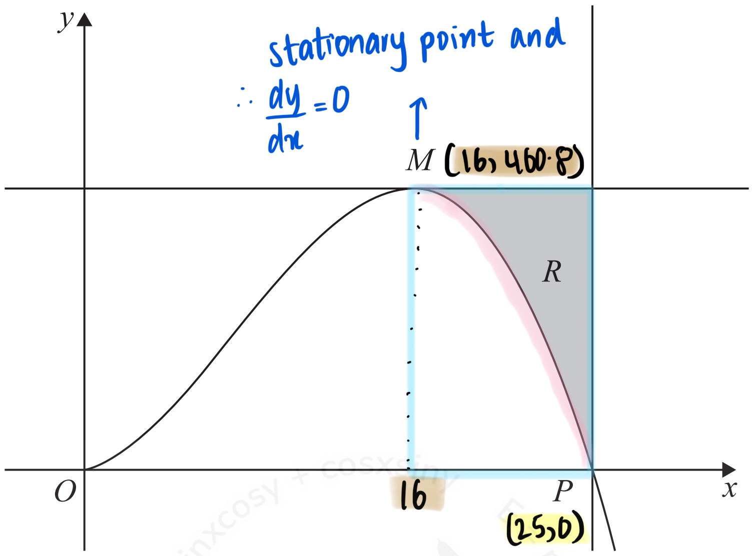

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2\left(5-\sqrt{x}\right)}{5} \qquad x \geqslant 0$$

The curve has a turning point at the point M, as shown in Figure 3.

(a) Using calculus, find the coordinates of M.

(5)

The curve crosses the x-axis at the point P, as shown in Figure 3.

(b) Use algebra to find the x coordinate of P.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x-axis and the line through P parallel to the y-axis.

(c) Use algebraic integration to find the area of R, giving your answer to one decimal place.

(5)

a) 1 Rewrite the equation of the curve and then differentiate

$y = \frac{9x^2(5 - x^{1/2})}{5}$

$= 9x^2 - \frac{9}{5}x^{5/2}$

$\frac{dy}{dx} = 18x - \frac{9}{2}x^{3/2}$

General Differentiation:
$\frac{d}{dx} a^n = na^{n-1}$

Question 9 continued

② M is the maximum point of the curve and therefore, is when $\frac{dy}{dx}=0$

$18x - \frac{9}{2}x^{3/2} = 0$

$36x - 9x^{3/2} = 0$

$36x = 9x^{3/2}$

$\frac{36}{9} = \frac{x^{3/2}}{x}$

$\therefore x^{1/2} = 4$

$x = 16$

③ Now we can simply subtitle our x value into the original equation of the curve to find y

$y = \frac{9(16)^2(5-\sqrt{16})}{5} =$

$y = 460.8$

$\therefore M = (16, 460.8)$

b) Point P is when the curve crosses the x axis and therefore is when y=0

$\therefore 0 = \frac{16x^2(5-\sqrt{x})}{5}$

$0 = 16x^2(5-\sqrt{x})$

$\therefore 5 - \sqrt{x} = 0$

$5 = \sqrt{x}$

$x = 5^2$

$= 25$

$\therefore$ x coordinate of $P = 25$

c) Area of R = Area of rectangle - Area under curve

① Let's find the area of the rectangle

$(25-16) \times 460.8 = \frac{20736}{5}$

$\therefore$ Area of rectangle $= \frac{20736}{5}$

P 7 9 5 9 1 A 0 2 7 3 2

Question 9 continued

(2) We can integrate the equation of the curve to find the area underneath it

$$\int_{16}^{25} 9x^2 - \frac{9}{5}x^{5/2}$$

$$\frac{9x^3}{3} \quad - \frac{9/5\, x^{7/2}}{7/2}$$

$$= \left[3x^3 - \frac{18}{35}x^{7/2}\right]_{16}^{25}$$

$$= \left(3(25)^3 - \frac{18}{25}(25)^{7/2}\right) - \left(3(16)^3 - \frac{18}{25}(16)^{7/2}\right)$$

$$= 6696.428 - 3861.9428$$

Area under graph $= 2834.4852$

(3) Area of R = Area of rectangle - Area under curve

$$4147.2 - 2834.4825$$

$$= 1312.7148$$

$\therefore$ Region $R = 1312.7$

Question 9 continued

(Total for Question 9 is 12 marks)

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos\theta\left(3\tan\theta + \frac{2}{\tan\theta}\right) \equiv \sin\theta + \frac{2}{\sin\theta} \qquad \theta \neq \frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$\cos x\left(3\tan x + \frac{2}{\tan x}\right) = 4\sin x - 5$$

giving your answers to 3 significant figures.

(4)

a) (1) First, let's change all of our tan values into $\frac{\sin\theta}{\cos\theta}$

$\cos\theta\left(3\frac{\sin\theta}{\cos\theta} + 2\frac{\sin\theta}{\cos\theta}\right)$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= 3\sin\theta + \frac{2\cos^2\theta}{\sin\theta}$

(2) Now we can can use the identity $\cos^2\theta = 1 - \sin^2\theta$

$3\sin\theta + \frac{2\cos^2\theta}{\sin\theta}$

$\cos^2\theta + \sin^2\theta = 1$

$3\sin\theta + \frac{2(1-\sin^2\theta)}{\sin\theta}$

Rewrite as

$3\sin\theta + \frac{2-2\sin^2\theta}{\sin\theta}$

$= 3\sin\theta + \frac{2}{\sin\theta} - \frac{2\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}}$

$= 3\sin\theta + \frac{2}{\sin\theta} - 2\sin\theta$

$= \sin\theta + \frac{2}{\sin\theta}$

P 7 9 5 9 1 A 0 3 0 3 2

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Question 10 continued

b) ① We can rewrite the LHS as $\sin x + \frac{2}{\sin x}$ and then solve!

$$\sin x + \frac{2}{\sin x} = 4\sin x - 5$$
$(\times \sin x)$ $(\times \sin x)$

$$\sin^2 x + 2 = \sin x(4\sin x - 5)$$
$$\sin^2 x + 2 = 4\sin^2 x - 5\sin x$$
$$\therefore 0 = 3\sin^2 x - 5\sin x - 2$$

② Now we can factorise and solve for x in radians

$$(\sin x - 2)(3\sin x + 1) = 0$$
$\therefore \sin x = 2$ or $\sin x = -\frac{1}{3}$

$\times$

$x = 0.3398\ldots$

$= 0.340$

as the range is $0 < x < 2\pi$

solution 1 $\rightarrow x = \pi + 0.340 = 3.482$

solution 2 $\rightarrow x = 2\pi - 0.34 = 5.943$

$\therefore x = 3.48,\ x = 5.94$

Question 10 continued

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS